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Uncertainty in Quali-Quantitative Response of a Natural Catchment on a Daily Basis

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Research strategy

- Aim of this study was the reproduction of the quali-quantitative response of a natural catchment on a daily basis

- The study was carried out linking a conceptual rainfall-runoff model to a model of nonpoint source pollutants (NPSP)

- This study explores how the limitations inherent in the modelling processes can be reflected in the estimation of predictive uncertainty
The hydrological response of the catchment has been modeled by means of the conceptual rainfall-runoff model IHACRES (Identification of Hydrographs And Components from Rainfall, Evapotranspiration and Streamflow Data) (Jakeman et al., 1990)

Non-linear loss module

\[ u(t) = s(t)^p \cdot r(t) \]

\[ s(t) = \frac{r(t)}{c} + \left( 1 - \frac{1}{\tau_w[T(t)]} \right) \cdot z^{-1} \cdot s(t) \]

\[ \tau_w[T(t)] = \tau_w e^{[0.062 \cdot f \cdot (20 - T(t))]} \]

where:
- \( u(t) \) effective rainfall;
- \( r(t) \) total rainfall;
- \( s(t) \) storage index (0<\( s_k <1 \));
- \( p \) exponent of a power-law;
- \( T \) air temperature;
- \( \tau_w \) basin drying time constant;
- \( f \) temperature modulation factor;
- \( c \) volume-forcing constant;
Conceptual modelling of hydrological response

Linear unit hydrograph module

Linear combination of two components describing the main runoff components: surface flow and subsurface flow

Two parallel elements, a linear channel corresponding to the ‘quick’ component of the total streamflow and a linear reservoir corresponding to the ‘slow’ component of the total streamflow.

\[ h(t) = c^q(t) \cdot \delta(t) + c^s(t) \exp(-\lambda t) \]

where:

\( h(t) \) effective rainfall;
\( c^q(t) \) quick flow volumetric throughput;
\( c^s(t) \) slow flow volumetric throughput;
\( \delta(t) \) Dirac delta function;
\( \lambda \) inverse of the time constant for the reservoir, \( \tau^s \);
Conceptual model of unit mass response function

The qualitative model deals with a conceptual form of the unit-mass response function of NPSP runoff to discharged water volume. It connects flow discharges to concentrations of pollutants, as nitrates (N-NO₃) and orthophosphates (P-PO₄).

It is possible to link directly unit hydrograph concepts to pollutant-water runoff via definition of a **unit-mass response function** (UMRF).

Mass flux against time in response to a rainfall event of unit intensity and duration uniformly distributed over the catchment.
The UMRF has been obtained by integration of the mass balance equations for the two elements (a canal and a reservoirs in parallel). In terms of UMRF, the $x^{th}$ pollutant load [g/s] is:

\[
\begin{align*}
    p_{tot} &= p_x(q) + p_x(s) \\
    p_x(q) &= C_{E,x} \cdot q(q) \\
    p_x(s) &= C_{E,x} \cdot q(s) \cdot \left( 1 - \frac{\lambda}{\lambda + h_x(s)} \right)
\end{align*}
\]

where:

- $p_x(q)$ $p_x(s)$ pollutant flow rate due to both, quick and slow, components of runoff;
- $C_{Ex}(q)$ $C_{Ex}(s)$ equilibrium concentrations for discharge rates and for the $x^{th}$ pollutant;
- $q_x(q)$ $q_x(s)$ quick surface runoff and slow subsurface runoff;
- $h_x(s)$ mass transfer coefficient for the slow component and for the $x^{th}$ pollutant;
- $\lambda$ storage coefficient for the slow component;
Parameters estimation

Non-linear loss module

- $\rho$: exponent parameter law
- $c$: volume-forcing constant
- $\tau_w$: basin drying time constant
- $f$: temperature modulation factor

Linear UH module

- $c^{(q)}$, $c^{(s)}$: quick and slow flow volumetric throughputs
- $\tau^{(s)}$: slow flow time constant

UMRF Model

- $C_{E,x}^{(q)}$, $C_{E,x}^{(s)}$: equilibrium concentrations for discharge rates and for the $x^{th}$ pollutant
- $h_x^{(s)}$: mass transfer coefficient for the slow component and for the $x$th pollutant
Parameters estimation

An analysis of model response in terms of prediction of the observed data has been made for each roughness distribution.

- It is a Monte Carlo based technique that allows for the concept of **equifinality** of parameter sets in the evaluation of modelling uncertainty;

- The performance of individual parameter sets are characterised by a likelihood weight, computed by comparing predicted to observed responses using some kind of likelihood measure. The model efficiency has been evaluated by the *Nash and Sutcliffe Efficiency Criterion*:

\[
L(\theta_i/Y) = \left(1 - \frac{\sigma^2}{\sigma_{\text{obs}}^2}\right) \quad \sigma^2 > \sigma_{\text{obs}}^2
\]

where:
- \(\sigma_i^2\) error variance
- \(\sigma_{\text{obs}}^2\) observed variance
- \(\theta_i\) parameter vector
- \(Y\) set of observation

\[\theta_i = [p, c, \tau_w, f, c^{(s)}, c^{(q)}, \tau^{(s)}]\] IHACRES

\[\theta_{i,x} = [C_{E,x}^{(q)}, C_{E,x}^{(s)}, h_x^{(s)}]\] UMRF model
Case study

The Nocella catchment with an area of 99.0 km² is an agricultural and urbanised catchment located in the north-western part of Sicily, Italy.

- **Equipment:**
  Nocella a Zucco gauging station

- **Available data:**
  Database of continuous measures of daily discharges and contemporary nitrogen and phosphorus concentrations on daily basis
Parameter estimation

A Monte Carlo procedure was used to generate 10000 sets of parameters for both models, each parameter value being drawn within ranges thought feasible for the Nocella catchment.

Simulations were performed for comparison with the measured daily flows (rainfall-runoff model) and measured pollutants discharges (qualitative model).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>c – volume-forcing constant (mm)</td>
<td>0.001</td>
<td>0.05</td>
</tr>
<tr>
<td>$t_w$ – basin drying time const. (days)</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>f – temp. modulation factor ($^\circ$C$^{-1}$)</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>$c^{(q)}$ – quick flow volumetric throughput</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$c^{(s)}$ = slow flow volumetric throughput</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$t^{(s)}$ – slow flow time const. (days$^1$)</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>p – power-law parameter</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$C_{E,x}^{(q)}$ $C_{E,x}^{(s)}$ – equilibrium concentr.</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>$h_x^{(s)}$ – mass transfer coefficients</td>
<td>1</td>
<td>30</td>
</tr>
</tbody>
</table>

**HYDROLOGICAL INPUT DATA:**

Daily series from May 1977 to April 1991 of rainfall and air temperature, spatially averaged over the catchment, and discharge data measured at the Nocella at Zucco gauging station.

**QUALITATIVE INPUT DATA:**

Continuous measures of daily discharge and contemporary nitrogen (N-NO$_3$) and phosphorus (P-PO$_4$) concentrations carried out at Nocella at Zucco gauging station site in March 2000.
A sensitivity analysis has been carried out in order to choose an initial condition $\theta$ belonging to the feasible region of parameters.

Using a Monte Carlo procedure 1000 values of the parameters were randomly generated with a uniform distribution.

The likelihood measure $L(\theta_i/Y)$ was evaluated for each parameter set and plotted versus the parameter values.
Parameter estimation for the IHACRES model

Simulations that achieve a likelihood value less than zero are rejected as non-behavioural. The remaining are rescaled between 0 to 1 in order to calculate the cumulative distribution of the predictive variables. Cumulative distributions of $c^{(q)}$, $c^{(s)}$, and $\tau^{(s)}$ represent the input variables in the qualitative model.
Parameter estimation for the UMRF model

- Using a Monte Carlo procedure 10000 uniform random sets of parameters are used to perform model simulation for the $x^{th}$ pollutant considered.
- The likelihood measure $L(\theta_i/Y)$ was evaluated for each parameter set and plotted versus the parameter values.
Parameter estimation for the UMRF model

- Simulations that achieve a likelihood value of zero are rejected as non-behavioural.
- Following the rejection of non-behavioural simulations, the weights rescaled between 0 and 1 have been applied to their respective model pollutant discharges to give a cumulative distribution of pollutant discharges at each time step, from which the chosen discharge quantiles, 5 and 95%, have been calculated to represent the model uncertainty
Conclusions

- A conceptual approach has been here presented to model the nonpoint source pollution transfer in a Mediterranean natural catchment.

- The modelling strategy was directed to a simple conceptual model because it provides good predictive accuracy in ephemeral streams, especially when only few and irregularly data are available.

- Daily basis modelling strategy is necessary for detailed reproduction of diffuse pollution, especially in semiarid environment.

- The GLUE approach focuses on the issues of the quali-quantitative model prediction, uncertainty and sensitivity inherent in the predictions of hydrological and nutrient characteristics.

- GLUE provides an estimate of the likelihood of the model given the observations and includes the effects of model structural error implicitly.