United Nations
Educational, Scientific and
Cultural Organization

International
Hydrological Programme
of UNESCO

Uncertainties in the ‘monitoring-conceptualisation-modelling’ sequence of catchment research

11th Conference of the Euromediterranean Network of Experimental and Representative Basins (ERB)
Luxembourg, 20 – 22 September 2006

Convened by:
Public Research Center – Gabriel Lippmann (Luxembourg) and ERB, UNESCO/IHP (NE FRIEND Project 5)

PROCEEDINGS

Edited by L. Pfister and L. Hoffmann

IHP-VI | Technical Documents in Hydrology | No. 81
UNESCO, Paris, 2007

Published in 2007 by the International Hydrological Programme (IHP) of the United Nations Educational, Scientific and Cultural Organization (UNESCO)
1 rue Miollis, 75732 Paris Cedex 15, France

IHP-VI Technical Document in Hydrology N°81
UNESCO Working Series SC-2007/WS/54

Publications in the series of *IHP Technical Documents in Hydrology* are available from:

IHP Secretariat | UNESCO | Division of Water Sciences
1 rue Miollis, 75732 Paris Cedex 15, France
Tel: +33 (0)1 45 68 40 01 | Fax: +33 (0)1 45 68 58 11
E-mail: ihp@unesco.org
http://www.unesco.org/water/ihp

(SC-2007/WS/54)

Printed in UNESCO's workshops
Paris, France

RAINFALL–STREAMFLOW MODELS FOR UNGAUGED BASINS: UNCERTAINTY DUE TO MODELLING TIME-STEP

I.G. Littlewood

Centre for Ecology and Hydrology, Wallingford, OXON, OX10 8BB, United Kingdom.
Corresponding author: Ian Littlewood, email: ianlittlewood505@btinternet.com

ABSTRACT

The extent to which the five parameters of a unit-hydrograph-based rainfall–streamflow model vary with modelling time-step is demonstrated for a 10.6 km^2 catchment in Wales. As the data time-step decreases from 24 hours to one hour, the calibrated parameters change by between 52% and 81%. The impact of time-step-dependent model parameters on the uncertainty in statistical relationships linking a model parameter and catchment properties is discussed. A simple method is described for normalising the parameters to give time-step-independent values. The results are discussed in terms of model parameter regionalisation towards estimation, from rainfall, of continuous streamflow in ungauged (flow) basins. Possible future work is outlined to compare the normalised model parameters presented in the paper with similar results from a modelling methodology presented by other authors that yields time-step-independent model parameters directly from analysis of discrete data.

Keywords: continuous rainfall-streamflow modelling, regionalisation, uncertainty, ungauged basins, unit hydrographs

Introduction

A popular approach to systematic estimation of river flows from rainfall in ungauged (flow) basins, regionally or nationally, has four main steps (omitting finer detail in the interests of brevity). In Step 1, a rainfall–streamflow model (*m* parameters) is calibrated for many (*n*) catchments. Statistical relationships (sample size *n*) are established in Step 2 between the *m*th model parameter and a few well-chosen catchment properties such as stream density, slope, etc. A different set of catchment properties might be used for each of the *m* statistical relationships. In Step 3, rainfall–streamflow model parameters for ungauged (flow) catchments are estimated using the regionalisation relationships established in Step 2. Flow in an ungauged catchment is estimated in Step 4 from rainfall and the rainfall–streamflow model parameters estimated from catchment properties in Step 3. Sefton and Howarth (1998) give an example of applying Steps 1 to 4.

Sources of uncertainty in a regionalisation scheme derived by Steps 1 to 4 include rainfall and streamflow measurement errors, the structure of the rainfall–streamflow model and uncertainties in its parameters, choice of catchment properties, and uncertainties in the regionalisation equations in Step 2.
The International Association of Hydrological Sciences (IAHS) Prediction in Ungauged Basins (PUB) Decade (http://pub.iwmi.org) has ‘reduction of predictive uncertainty’ as a cross-cutting objective, so all sources of uncertainty in regionalisation schemes are targets for better understanding and reduction of their impacts. In the context of gauged and ungauged UK catchments, the author has already discussed some aspects of uncertainty in (a) regionalised unit hydrographs for flood event hydrology and (b) the parameters of a continuous simulation rainfall-streamflow model (Littlewood, 2003; 2004). This paper addresses an aspect of uncertainty in rainfall-streamflow model parameter regionalisation that has, to date, received little attention in the hydrological research literature, i.e. the dependency of rainfall–streamflow model parameters on the data time-step employed for model calibration.

Constraints on the resources available for deriving regionalisation schemes for systematic estimation of continuous hydrographs at ungauged sites (e.g. Sefton and Howarth, 1998) have often led to the use of readily available daily data for calibrating rainfall–streamflow models in Step 1 (e.g. the use of flow data from the UK

National River Flow Archive, http://www.ceh.ac.uk/data/nrfa/index.html). A given model structure calibrated using a given data time-step will, of course, have different parameters for different catchments exhibiting different rainfall-streamflow dynamics. However, a discrete-time model calibrated for a particular catchment will yield different parameters according to the data time-step employed. When a common data time-step is used for all catchments for which rainfall-streamflow models are calibrated (Step 1), a source of uncertainty therefore arises in the statistical relationships between a model parameter and catchment properties (Step 2). As a contribution to PUB via its Top-Down modelling Working Group (http://www.stars.net.au/tdwg/), this paper demonstrates the extent to which the parameters of a rainfall-streamflow model, for a small research catchment, vary with the time-step of the data used for model calibration. A simple method is then outlined for normalising the model parameters to render them independent of the data time-step used for model calibration. The paper discusses implications of the data time-step-dependency of rainfall-streamflow model parameters in the context of uncertainty in flows in ungauged basins estimated systematically from catchment properties.

The catchment and modelling scheme

The Wye at Cefn Brwyn is a 10.6 km^2, predominantly open moorland, catchment draining the headwaters of the River Wye in mid-Wales (Fig. 1). It is one of the wettest gauged basins in England and Wales; mean annual rainfall is about 2490 mm, of which about 87% leaves the catchment as streamflow (NERC, 2003). The catchment is one of the Plynlimon research basins operated by the Centre for Ecology and Hydrology (e.g. Brandt et al., 2004; Robinson and Dupeyrat, 2005).

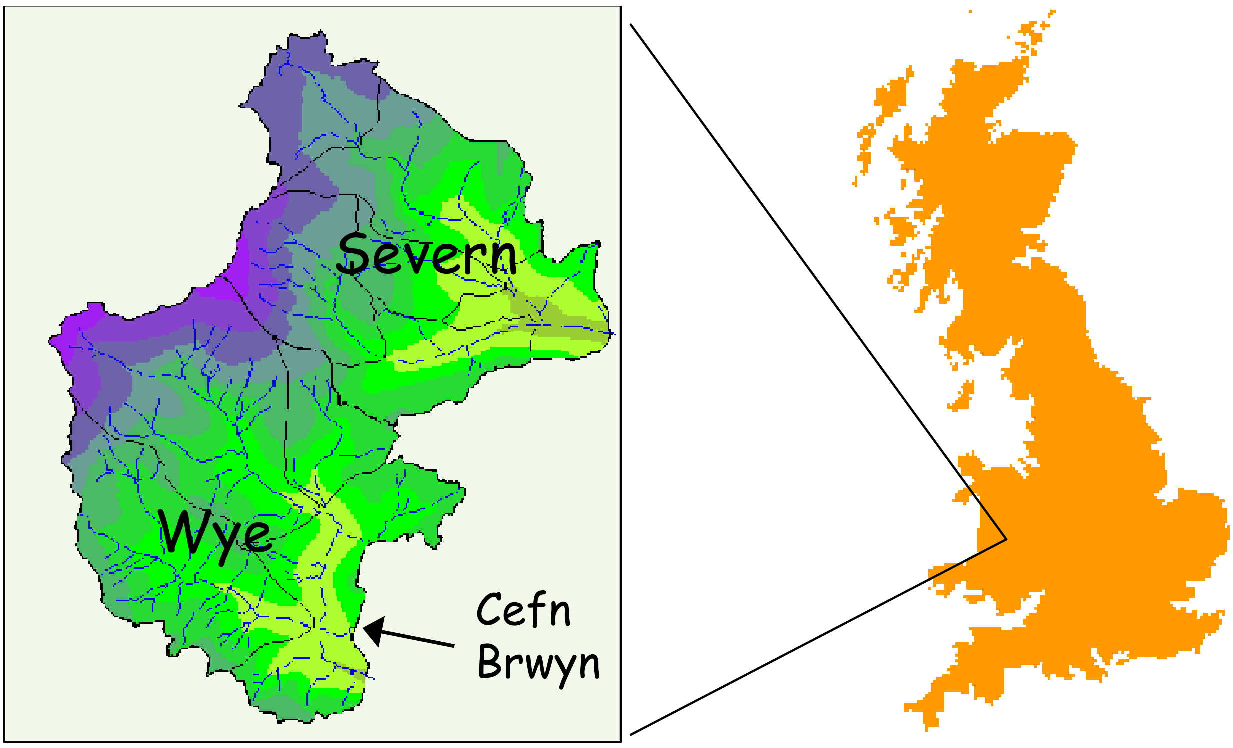

Fig. 1: Location of the Wye at Cefn Brwyn.

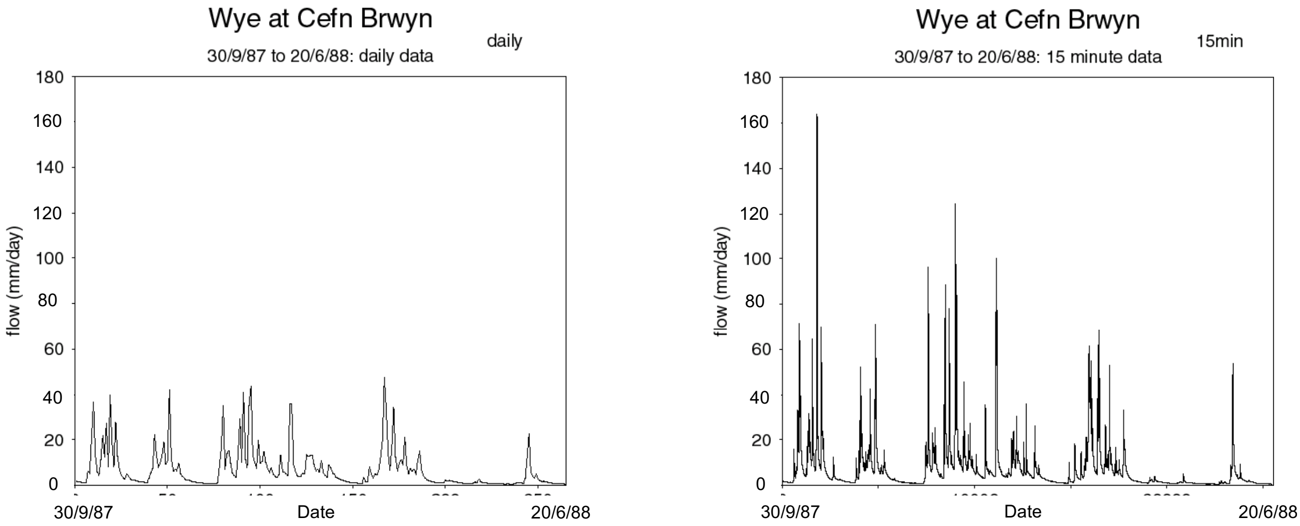

Fig. 2: Wye at Cefn Brwyn 24-hourly (left) and 15-minute (right) hydrographs.

Fig. 2 shows daily (265 points) and 15-minute (25,440 points) hydrographs from 30 September 1987 to 20 June 1988, indicating the extent to which a daily time-step hydrograph masks the highly dynamic flow regime at Cefn Brwyn. The available hydrometric data comprised 15-minute flow data and hourly catchment rainfall data, from which 1-, 2-, 4-, 6-, 12- and 24-hourly time-step rainfall-streamflow datasets were prepared. Using each dataset in turn, a rainfall-streamflow model was calibrated over the period from 6 December 1987 to 2 July 1988. The model employed was the spatially-lumped, unit-hydrograph-based, IHACRES (Identification of unit Hydrographs And Component flows from Rainfall, Evaporation and Streamflow data) model (Jakeman et al., 1990; Littlewood and Jakeman, 1992; 1994). The software used was IHACRES Classic Plus (Croke et al., 2006). Fig. 3 is a schematic of the model, giving outline descriptions of its six dynamic response characteristics (DRCs), f, c, τ_w, $\tau^{(q)}$, $\tau^{(s)}$ and $v^{(s)}$ (dimensions in square brackets). At time-step k, rainfall (r_k) and air temperature (t_k) are input to the loss module, which produces effective rainfall (u_k). The unit hydrograph module produces streamflow (x_k) from u_k. When f is set to zero, it plays no part in the model and air temperature (t_k) is not used. The models in this paper were thus constrained to have five DRCs.

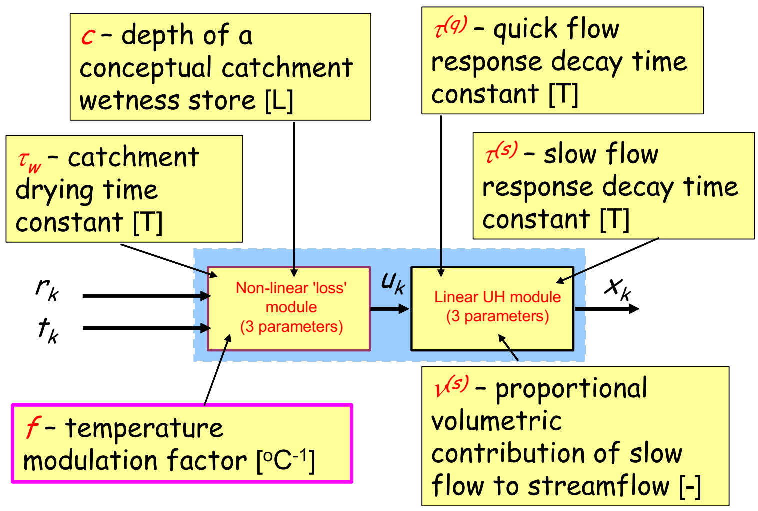

Fig. 3: IHACRES model structure and dynamic response characteristics (DRCs).

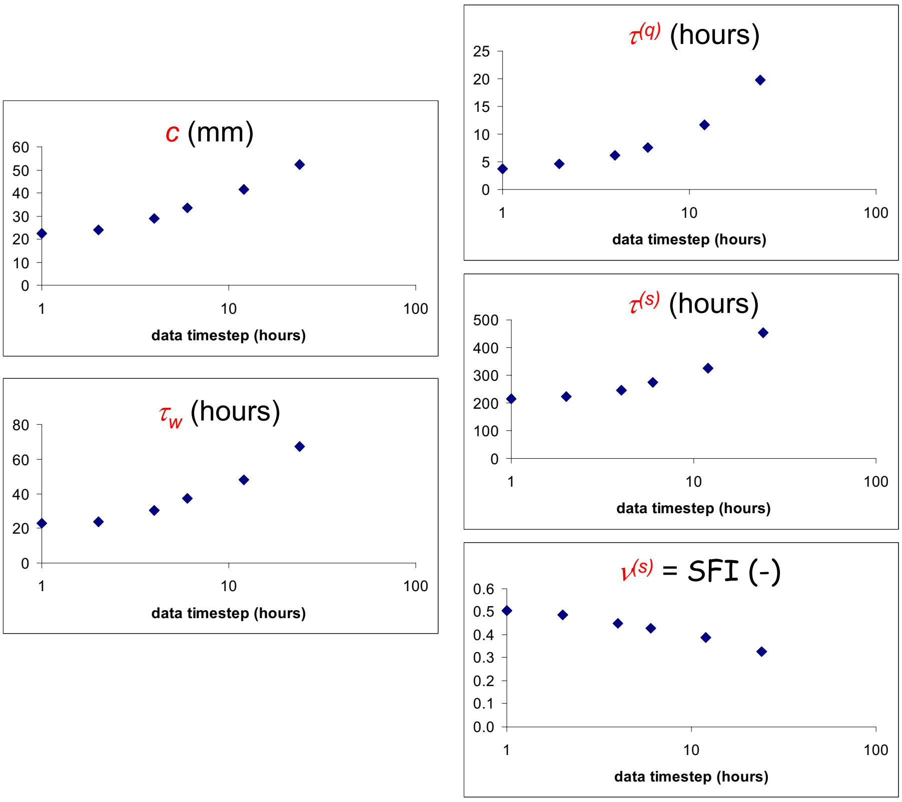

Fig. 4: Dynamic response characteristics against logarithm of modelling time-step.

Results

Fig. 4 shows each of the five DRCs, c, τ_w, $\tau^{(q)}$, $\tau^{(s)}$ and $v^{(s)}$ plotted against the logarithm of the data time-step (1-, 2-, 4-, 6-, 12- and 24-hours) used for corresponding model calibrations. The figure shows a systematic relationship for each DRC between its modelled value and the data time-step employed for model calibration. As the data time-step decreases from 24 hours to 1 hour, DRCs c, τ_w, $\tau^{(q)}$ and $\tau^{(s)}$ decrease by 66%, 81%, 52% and 57% respectively, and $v^{(s)}$ increases by 55%; substantial changes for each DRC. Other catchments, having different streamflow dynamics, will exhibit different relationships between each DRC and data time-step. Therefore, the use of a given data time-step for all n rainfall-streamflow models calibrated in Step 1, towards establishing a model parameter regionalisation scheme, is inappropriate and will cause some of the uncertainty in statistical models linking DRCs and catchment properties established in Step 2. What is required in order to eliminate, or at least reduce, this component of uncertainty is a method of normalising the DRCs so they are independent of data time-step. DRCs corresponding to a data time-step of zero would fulfil this objective (see next section). Although the problem has been illustrated here using IHACRES, the parameters of other conceptual-metric (Wheater et al., 1993) rainfall–streamflow models calibrated using temporally discrete data are also likely to be dependent on the data time-step employed (but see later discussion of this point).

Each of the models (using 1-, 2-, 4-, 6-, 12-, and 24-hourly data) was calibrated on the basis of a trade-off between a high value for a coefficient of determination (D) given by equation (1) and a low average relative parameter error (*ARPE*) for the unit hydrograph component of the model, as described by Jakeman et al. (1990) where details of how *ARPE* is calculated are given. In equation (1) Q_0 and Q_m are observed and modelled flow respectively. As the data time-step decreases from 24 hours to 1 hour, D increases from 0.895 to about 0.907, and *ARPE* decreases from 0.142 to 0.001, giving increasing confidence in the estimated DRCs as data time-step decreases.

$$D = 1 - \frac{\Sigma(Q_0 - Q_m)^2}{\Sigma(Q_0 - \overline{Q_0})^2} \qquad \text{Eq. 1}$$

Rainfall–streamflow model parameter normalisation

The curves for c, τ_w and $\tau^{(s)}$ in Fig. 4 reach asymptotes at a data time-step of 1 hour, and for $\tau^{(q)}$ and $v^{(s)}$ nearly so, suggesting that modelling at sub-hourly data time-steps is required to reach asymptotes for those DRCs for the Wye at Cefn Brwyn (this requires additional data preparation and has not been attempted for this short paper). The asymptotes are effectively values of the DRCs at a data time-step of zero and, being thus independent of data time-step, provide a superior characterisation of rainfall–streamflow dynamic behaviour than time-dependent DRCs. From Fig. 4, normalised DRCs, $c^{'}$, $\tau_w^{'}$, $\tau^{(q)'}$, $\tau^{(s)'}$ and $v^{(s)'}$ for the Wye at Cefn Brwyn are, respectively, about 20 mm, 20 hours, 3 hours, 200 hours and 0.51 (dimensionless). The DRCs $\tau^{(q)'}$, $\tau^{(s)'}$ and $v^{(s)'}$ are instantaneous unit hydrograph parameters for total streamflow and, by analogy, the DRCs $c^{'}$ and $\tau_w^{'}$ are instantaneous loss module parameters. Taken together, the five normalised DRCs give an instantaneous rainfall–streamflow model for the Wye at Cefn Brwyn.

Discussion

The structure of the unit hydrograph module employed here comprises two linear stores acting in parallel, representing dominant quick- and slow-response, catchment-scale, streamflow generation processes respectively. Experience with IHACRES has shown that, from information in the rainfall–streamflow records typically available at a sufficient number of locations for regionalisation studies, two linear stores in parallel are usually an appropriate unit hydrograph structure (e.g. better than two similar stores in series). It is possible, however, that a more detailed unit hydrograph structure still having conceptual appeal (e.g. three stores in parallel, requiring an additional two model parameters) might be identifiable from small time-step data (e.g. hourly) for the Wye at Cefn Brwyn. Investigation of this possibility was considered to be unimportant for the theme of this paper and remains to be undertaken.

The first application of the IHACRES methodology was to two small (<1 km^2) catchments in Wales located about 40 km south of Plynlimon (Littlewood, 1989; Jakeman et al., 1990), using hourly data. The flow regimes of those small basins, situated in the headwaters of the Llyn Brianne catchment, are not too different to the flow regimes observed in the headwaters of the Wye. The work presented in the current paper (for the Wye at Cefn Brwyn), supports the title " Computation of the *instantaneous* unit hydrograph …" (italics added) of the Jakeman et al. (1990) paper, because the unit hydrograph DRCs for the Llyn Brianne catchments were derived using hourly data and are likely to be good approximations of the asymptotic values (Fig. 4) that would have been obtained if modelling had been undertaken using a range of sub-daily data time-steps. Data for the Llyn Brianne catchments are not readily available to the author, so this idea has not been tested here.

However, in the wettest regions of the United Kingdom the dynamics of many headwater flow regimes can be severely masked by a daily mean flow hydrograph, in which case modelling at sub-daily time-steps will be required to identify time-independent DRCs. Indeed, this will be the case whenever the quick-flow response time is close to, or less than, one day. The unit hydrograph DRCs $\tau^{(q)}$, $\tau^{(s)}$ and $v^{(s)}$ derived using daily data for UK catchments having areas of tens to hundreds of km^2 and highly dynamic flow regimes (e.g. Sefton and Howarth, 1998; Littlewood, 2003) may not be optimal for subsequent parameter regionalisation, because they are unlikely to be good approximations of time-independent DRCs. Loss module parameters c and τ_w, (and f, when included) for such catchments will also be sub-optimal. Rainfall–streamflow model parameter regionalisation should consider carefully the possible impact, in terms of uncertainties, of using a single modelling time-step for all catchments. Dependency of model parameters on modelling time-step should be assessed in each case and, when necessary, the model parameters should be normalised to be time-independent.

In order to estimate flows in an ungauged catchment using discrete-time rainfall data, a set of normalised DRCs ($f^{'}$, $c^{'}$, $\tau_w^{'}$, $\tau^{(q)'}$, $\tau^{(s)'}$ and $v^{(s)'}$) will need to be de-normalised to DRCs corresponding to the time-step of the available rainfall data. How to execute this back-transformation for ungauged catchments is beyond the scope of the current paper but there will, inevitably, be uncertainty associated with this step. It remains to be established whether or not the uncertainty in flows estimated via normalised DRCs is less than if time-dependent DRCs were used.

The method for estimating time-independent (normalised) DRCs presented in this paper, based on analysis of data from just one catchment, required preparation of datasets at different time-steps, and was demanding in terms of repetitive application of the modelling software. With (a) enhancements to database facilities, e.g. making high temporal resolution (sub-daily) data available and (b) further development of the modelling software to automate compilation of different time-step datasets and management of the repetitive modelling process using those different time-step datasets, it would be possible to implement the method systematically.

Further work is required to investigate whether or not the simple method for normalising IHACRES model parameters presented in this paper, for the Wye at Cefn Brwyn, will work well for other catchments (Littlewood and Croke, submitted). At least one other method of model parameter normalisation should also be evaluated. Young and Garnier (2006) describe data-based mechanistic (DBM) methods for deriving continuous-time models from discrete-time input data, where the calibrated model parameters are independent of the time-step of the input data. Since IHACRES is a special case of a DBM model, an extension of the work described above would be to apply DBM methods to the Wye at Cefn Brwyn and other catchments, and to compare the estimates of time-step-independent model parameters thus obtained with the normalised IHACRES DRCs presented in this paper.

Concluding remarks

Rainfall-streamflow model parameters (DRCs) for the Wye at Cefn Brwyn, estimated by the IHACRES methodology, have been shown to vary substantially with the data time-step used for model calibration. At a data time-step of 1 hour, each relationship between a DRC and the logarithm of modelling time-step reaches, or approaches, an asymptote that reasonably can be expected to be a better characterisation of the catchment rainfall–streamflow dynamic behaviour than a DRC corresponding to a larger data time-step. For regionalisation studies the technique would have to be applied to many gauged catchments. It is expected that there will be a better statistical link between a normalised DRC and catchment properties than between a time-step-dependent DRC and catchment properties. Further work is required to test this idea. Further work is also required to (a) compare DRCs normalised as described in this paper with time-step-independent model parameters derived by continuous-time DBM models corresponding to IHACRES and (b) devise a method of back-transforming normalised (time-independent) model parameters to time-step-dependent model parameters for the estimation, from discrete rainfall data, of flows in ungauged catchments.

References

Brandt C., Robinson M., Finch J.W. (2004). Anatomy of a catchment: the relation of physical attributes of the Plynlimon catchments to variations in hydrology and water status. *Hydrology and Earth System Sciences*, 8: 345-354.

Croke B.F.W., Andrews F., Jakeman A.J., Cuddy S.M., Luddy A. (2006). IHACRES Classic Plus: a redesign of the IHACRES rainfall-runoff model. *Environmental Modelling and Software*, 21: 426-427.

Jakeman A.J., Littlewood I.G., Whitehead P.G. (1990). Computation of the instantaneous unit hydrograph and identifiable component flows with application to two small upland catchments. *Journal of Hydrology*, 117: 275-300.

Littlewood I.G. (1989). *The Dynamics of Acid Runoff from Moorland and Conifer Afforested Catchments Draining into Llyn Brianne, Wales*. PhD Thesis, University of Wales (Swansea), 338 pp.
Littlewood I.G. (2003). Improved unit hydrograph identification for seven Welsh rivers: implications for estimating continuous streamflow at ungauged sites. *Hydrological Sciences Journal*, 48: 743-762.
Littlewood I.G. (2004). Unit hydrographs and regionalization of United Kingdom river flows: comments on some estimation uncertainties. In: C. Pahl-Wostl, S. Schmidt, A.E. Rizzoli, A.J. Jakeman (Eds), *Complexity and Integrated Resources Management, Transactions of the 2nd Biennial Meeting of the International Environmental Modelling and Software Society*, Manno, Switzerland, iEMSs, 2004, 3: 1153-1158.
Littlewood I.G., Croke B.F.W. (submitted). Towards an instantaneous rainfall-streamflow model for ungauged basins: parameter dependency on data time-step. *Hydrological Sciences Journal*.
Littlewood I.G., Jakeman A.J. (1992). Characterisation of quick and slow streamflow components by unit hydrographs for single- and multi-basin studies. In : M. Robinson (Ed.), *Proceedings of the Fourth General Assembly of the European Network of Experimental and Representative Basins*, Oxford, September 29 - October 2, published as Institute of Hydrology Report 120.
Littlewood I.G., Jakeman A.J. (1994). A new method of rainfall-runoff modelling and its applications in catchment hydrology. In: P. Zannetti (Ed.), *Environmental Modelling (Volume II)*, Computational Mechanics Publications, Southampton, UK: 143-171.
NERC (2003). *Hydrological Data UK: Hydrometric Register and Statistics 1996-2000.* Centre for Ecology and Hydrology, Natural Environment Research Council, 208 pp.
Robinson M., Dupeyrat A. (2005). Effects of timber harvesting on streamflow regimes in the Plynlimon catchments, mid-Wales. *Hydrological Processes*, 19: 1213-1226.
Sefton C.E.M., Howarth S.M. (1998). Relationships between dynamic response characteristics and physical descriptors of catchments in England and Wales. *Journal of Hydrology,* 211: 1–16.
Wheater H.S., Jakeman A.J., Beven K.J. (1993). Progress and directions in rainfall – runoff modelling. In: A.J. Jakeman, M.B. Beck, M.J. McAleer (Eds.), *Modelling Change in Environmental Systems*, John Wiley & Sons Ltd. 99-130.
Young P.C., Garnier H. (2006). Identification and estimation of continuous-time, data-based mechanistic (DBM) models for environmental systems. *Environmental Modelling and Software*, 21: 1055-1072.